Net Real Financial Cost of Government (CFRN)

A methodological framework for measuring the effective real financing rate and its burden relative to GDP

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Methodological paper

Abstract

This paper proposes a standardized framework for measuring the ex post net real financial cost of government. The starting point is the distinction between the nominal budgetary cost of financing and its real economic cost: interest is a flow, whereas inflation, contractual indexation, and exchange-rate movements alter the real value of principal. The absolute Net Real Financial Cost (CFRN) is treated as an auxiliary variable. The principal outputs are two dimensionless measures: the Net Real Financial Cost Rate (TCFRN), which expresses net real cost per unit of interest-bearing financial liabilities, and CFRN/GDP, which expresses the real financing burden relative to annual economic capacity.

The literature contains several close antecedents. The IMF's operational deficit adjusts the conventional deficit for the inflation component of debt service; Buiter (1983) incorporates the erosion of nominal liabilities into balance-sheet accounting; Hall and Sargent (2011) decompose debt-to-GDP dynamics through debt returns, inflation, and growth; Reinhart and Sbrancia (2011, 2015) calculate a 'liquidation effect' as a negative real rate multiplied by domestic debt and expressed relative to GDP; and Hilscher, Raviv, and Reis (2022) develop a forward-looking, market-value measure of debt debasement through inflation. Accordingly, neither the effective real rate nor its product with the debt stock is a novel identity. The proposed contribution is one of standardization: a symmetric framework for positive and negative real costs, based on strict stock-flow consistency, explicit treatment of indexed and foreign-currency debt, distinction between institutional perimeters, a net financial-asset variant, and a reporting convention that places TCFRN and CFRN/GDP at the center of the analysis.

The method recommends accrual accounting, average liabilities consistent with recorded interest flows, the GDP deflator as the benchmark for comparisons of fiscal burden, and consumption-price measures as sensitivity checks. The two main outputs are linked by an exact identity—CFRN/GDP = TCFRN × liabilities/GDP—which allows the macroeconomic burden to be interpreted as the product of the effective real price of financing and the scale of exposure. The paper also distinguishes a contractual or accounting version, suitable for historical series and fiscal comparison, from a market-value version based on holding-period returns and valuation changes.

Keywords: public debt; interest; inflation; real financing cost; effective real interest rate; operational deficit; financial repression; public debt-to-GDP; public finance.

JEL codes: E31, E43, E62, H62, H63.

1. Introduction

Conventional measurement of the cost of public debt relies on nominal and budgetary indicators: interest paid, net interest, the effective cost of debt, interest relative to GDP, or interest relative to public revenue. These measures are indispensable for analyzing liquidity, budgetary pressure, and refinancing risk. Yet a public financial obligation is simultaneously a debt-service flow and a balance-sheet stock. When the liability is denominated in nominal terms, its real value changes even if its face value does not.

Inflation reduces the purchasing power of a domestic nominal obligation. Principal indexation can offset that erosion. Currency depreciation can increase, in domestic-currency terms, the principal of debt denominated in foreign currency. The real economic cost of carrying a debt portfolio therefore cannot be inferred solely from the nominal interest recorded in the budget.

The purpose of this paper is to propose a replicable measurement convention for that dimension. The central issue is not the absolute monetary amount of the adjustment—which depends on country size and currency unit—but two relative objects:

$$TCFRN_t = \text{net real financial cost / average base of interest-bearing financial liabilities}$$

$$\text{CFRNGDP}_t = \text{net real financial cost} / \text{GDP}_t$$

TCFRN answers a financial question: what net real cost has the public sector borne per unit of financial liability included within the chosen perimeter? CFRN/GDP answers a macroeconomic question: what burden does that real cost represent for the economy? The two are not independent indicators. When they share the same numerator and liability base, CFRN/GDP is exactly TCFRN multiplied by the liabilities-to-GDP ratio. This identity—real price times scale of exposure—is one of the organizing relationships of the framework.

2. What CFRN Measures—and What It Does Not

2.1. An ex post holding cost, not a cash-flow measure

CFRN is intended to measure the ex post real economic cost of carrying financial liabilities over a period. It does not replace cash interest expenditure. A Treasury can face a large cash outflow and, at the same time, record a low or negative real cost on inherited nominal debt when inflation exceeds the portfolio's effective nominal cost. Both statements can be simultaneously correct because they describe different dimensions.

2.2. Not a comprehensive measure of fiscal sustainability

The indicator does not incorporate the primary balance, real growth, new financing needs, contingent guarantees, or the full set of other stock-flow adjustments. A negative TCFRN therefore does not imply debt sustainability. It means only that the real financial component for the period, within the defined perimeter, was negative.

2.3. Not a normative welfare measure

The real erosion of nominal liabilities is a valuation gain for the debtor and a loss for the creditor. The indicator does not claim that inflation is socially desirable. Distributional costs, credibility effects, allocative distortions, real-wage effects, and risk-premium consequences lie outside the metric.

2.4. Not a measure of the marginal cost of new issuance

TCFRN is backward-looking and based on the effective cost of the outstanding portfolio. It should not be confused with the market yield on a new issue or with the marginal cost of refinancing. Because of duration and staggered maturities, these quantities can diverge for years.

3. Antecedents: Existing Measures and the Position of the Proposal

A publishable formulation of CFRN requires a precise account of its relationship to existing concepts. Inflation adjustment of public interest payments has a long tradition, and several institutions already use effective real rates on government debt. The defensible contribution of CFRN is therefore not a new algebraic identity, but a more explicit, symmetric convention for measurement and reporting.

3.1. Buiter (1983): from budget accounting to real public wealth

Buiter's starting point is broader than the measurement of interest: the conventional deficit is not, by itself, an adequate measure of the economic change in the public sector's position. His framework uses comprehensive wealth-and-income accounting in which government holds explicit and implicit assets and

liabilities. The relevant object is the change in real net worth, not merely the current difference between budgetary revenue and expenditure.

The reason is that budget flows omit capital gains and losses on pre-existing stocks. Buiter explicitly identifies three families of adjustments: changes in the real value of public assets, valuation changes in foreign-currency assets and liabilities, and inflationary erosion of nominal obligations. In an inflationary environment, government can report a large nominal deficit while the real value of its nominal debt falls substantially. A wealth-based analysis should therefore recognize the decline in the real value of those liabilities as a gain to the debtor.

For a closed economy with nominal debt, Buiter's intuition can be summarized as an adjustment proportional to inflation times the real stock of nominal obligations. If the Fisher effect operates and nominal rates rise roughly one-for-one with inflation, higher nominal interest payments are offset by a decline in the real value of principal. Buiter even presents deficit-adjustment exercises for the United States and the United Kingdom, and argues that public accounting should stop treating inflation as though it left the government's balance sheet unchanged.

The relationship with CFRN is therefore deep but partial. Buiter asks how public wealth changes and what that measurement implies for crowding out, financing, and monetary policy. CFRN deliberately asks a narrower question: what is the real financial cost of carrying a portfolio of liabilities over a period? It adopts from Buiter the need to recognize real valuation gains and losses, but excludes nonfinancial public wealth, future taxes, social entitlements, and other items that would turn a financing-cost measure into a comprehensive public-sector balance sheet.

3.2. Tanzi, Bléjer, and Teijeiro (1987): inflation, debt service, and the operational deficit

Tanzi, Bléjer, and Teijeiro study a problem that became particularly visible in high-inflation economies: two governments with the same primary expenditure and a similar real debt burden can display very different nominal deficits if one experiences much higher inflation. The reason is that part of nominal interest compensates the holder for the loss of purchasing power of principal and need not represent an equally large real transfer of income.

$$iD \approx rD + \pi D$$

This decomposition gives rise to the operational deficit. In its usual formulation, the portion of debt service attributable to inflation is subtracted from the conventional deficit; equivalently, only the real component of interest is added to the primary deficit. The intuition is that inflation compensation is economically akin to amortization of principal: it preserves the real value of the security rather than remunerating the creditor above that value.

The article does not, however, conclude that the operational deficit should always replace the conventional deficit. The authors show that both can provide incomplete signals. Nominal financing needs remain relevant for liquidity and refinancing even when part of the interest does not represent a real cost. Moreover, the estimated inflation-compensation component depends on the chosen price index, the structure of the debt, the ability to refinance debt service, and whether real rates are negative. Their conclusion is notably cautious: when inflation is high, both measures should be calculated and examined.

CFRN shares the real adjustment but changes the object being measured. It does not ask which fiscal deficit is 'correct'; it asks what the real financing cost is. A negative real rate is therefore neither an accounting anomaly nor a value that must be floored at zero: it means that the real reduction in the liability

exceeded the period's net carry. This change in object allows CFRN to treat positive and negative costs symmetrically without, by itself, making any claim about the overall fiscal stance.

3.3. Bléjer and Cheasty (1991): no single deficit measure answers every question

Bléjer and Cheasty broaden the methodological debate. Their contribution is not merely to refine the operational deficit, but to show that the phrase 'the fiscal deficit' conceals conceptual choices about cash versus accrual accounting, institutional perimeter, quasi-fiscal operations, assets, arrears, public enterprises, future commitments, and valuation. The appropriate measure depends on the economic question being asked.

This point is relevant to the rationale for CFRN. The fact that its ingredients—effective interest, inflation, the debt stock, and financial assets—are already known does not eliminate a distinct measurement problem. The methodological requirement is to define the object precisely, match stocks with flows, disclose valuation conventions, and limit interpretation to the phenomenon actually measured. CFRN belongs to that tradition: it does not seek a 'true deficit', but isolates a specific dimension of public finance that aggregate indicators combine with other components.

3.4. The effective real interest rate in IMF macrofiscal practice

The closest algebraic antecedent to TCFRN is the effective real interest rate on government debt. Macrofiscal practice distinguishes the market yield on a new issue from the effective cost of the existing portfolio. A common convention first calculates an effective nominal rate as interest for the period divided by a lagged or average debt stock, and then adjusts it for inflation. The IMF has explicitly used this concept to study how inflation and market rates pass through to debt service, defining the effective rate from interest expenditure relative to prior debt and examining its real counterpart.

$$i^{eff}_t = I_t / L_{t-1} \quad ; \quad r^{eff}_t \approx i^{eff}_t - \pi_t$$

This antecedent sharply limits any claim of originality for CFRN. In the elementary case of domestic nominal debt, with no relevant financial assets or indexation, TCFRN is approximately the effective real rate and, using the Fisher transformation, can coincide exactly under the same timing convention. The contribution of the framework is therefore not to discover $i - \pi$, but to establish a coherent standard for perimeter, stock-flow matching, indexation, currency denomination, asset netting, and the translation of cost per unit of debt into a macroeconomic burden through CFRN/GDP.

3.5. Reinhart and Sbrancia (2011, 2015): financial repression and the 'liquidation effect'

Reinhart and Sbrancia study how many governments reduced large debt stocks after the Second World War without relying exclusively on primary surpluses, growth, or default. Their central mechanism is financial repression: capital controls, explicit or implicit ceilings on interest rates, regulations that create captive demand for government securities, and other restrictions that allow nominal yields to remain below the levels that would prevail in fully liberalized markets.

When those low nominal rates are combined with inflation, ex post real returns on public debt can become negative. The authors call years in which the contractual real rate on government debt is negative 'liquidation years' and calculate a 'liquidation effect' that quantifies the equivalent saving or revenue accruing to the government. The logic is very close to multiplying the negative real rate by the debt stock and expressing the result as a percentage of GDP. Across their sample of twelve countries, they estimate average annual interest-cost savings on the order of 1–5 percent of GDP during 1945–1980, with negative real rates occurring frequently in advanced economies.

This is probably the closest antecedent to CFRN/GDP when the result is negative. The difference should not be overstated: the underlying economic operation is similar. The proposed contribution of CFRN is to generalize the measurement beyond episodes of financial repression and retain the full distribution of outcomes. A year with positive TCFRN is as informative as a year with negative TCFRN; the metric can be used continuously under inflation, price stability, or deflation. In addition, CFRN requires consistency with the full set of liabilities included, allows financial income to be netted, and establishes explicit rules for indexed debt, foreign currency, and consolidation.

3.6. Hall and Sargent (2011): the economic return on debt and market valuation

Hall and Sargent ask a different question: what economic return did U.S. federal debt generate, and how did nominal returns, inflation, and growth contribute to postwar debt-to-GDP dynamics? To answer it, they regard official interest payments as insufficient because a bondholder's return includes not only the coupon but also capital gains or losses arising from changes in the security's market price.

Their methodology uses a sequence of government budget constraints and maturity-specific holding-period returns. A rise in market rates can reduce the price of previously issued securities, producing a loss for the creditor and lowering the market value of the public liability even though the Treasury remains contractually obliged to repay the same principal at maturity. Their estimates therefore differ both conceptually and quantitatively from official interest figures.

This distinction is essential for CFRN. Contractual or accounting CFRN measures accrued cost under the terms of issuance and is the appropriate variant for long, comparable fiscal series. A market-based CFRN can incorporate holding-period returns and valuation changes, but it must be reported separately. Mixing the two bases would produce a difficult-to-interpret number: a fall in a bond's price lowers the market value of the liability, but does not automatically reduce the contractual amount the government must pay if the security is held to maturity.

3.7. Hilscher, Raviv, and Reis (2022; working paper 2014): prospective debasement, maturities, and expectations

Hilscher, Raviv, and Reis ask how much future inflation could reduce the real value of outstanding U.S. debt. Their object is not the cost observed in a past year, but the forward-looking distribution of potential real losses on the debt. They estimate it by combining the maturity structure of privately held nominal debt with risk-adjusted inflation distributions inferred from option prices.

Their analysis shows why the rule πD can be a poor prospective approximation. The capacity to debase debt depends on who holds the securities, when they mature, the persistence and volatility of inflation, and how much inflation was already anticipated and therefore embedded in prices. For the United States, they conclude that inflation alone is unlikely to reduce the fiscal burden by very large amounts under plausible scenarios: the relevant debt is concentrated at relatively short maturities and markets attach low probability to large, persistent inflation surprises. In 2017, they estimate that debasement exceeding only a few percentage points of GDP was highly unlikely.

The contrast with CFRN is both temporal and conceptual. Hilscher et al. ask how much debt might be debased in the future; ex post CFRN asks what the portfolio actually cost during the observed period. In a current-period measure, short-maturity debt still loses purchasing power while it remains outstanding; short duration matters because refinancing transmits expected inflation and market rates rapidly into future nominal carry. Duration is therefore central to a prospective exercise, but it should not be mechanically converted into an arbitrary discount on real erosion that has already occurred during the holding period.

Table 1. What question does each antecedent answer, and how does it relate to CFRN?

Work / approach	Question and method	Main conclusion	Relationship to CFRN
Buiter (1983)	How does public-sector net worth actually change? Comprehensive accounting of assets/liabilities and valuation gains.	The conventional deficit can be highly misleading; inflation reduces the real value of nominal liabilities.	Balance-sheet foundation. CFRN isolates financing cost and prioritizes the rate and the burden relative to GDP.
Tanzi, Bléjer, and Teijeiro (1987)	Which deficit is informative under high inflation? Operational deficit = primary deficit + real interest.	Inflation compensation resembles amortization; conventional and operational deficits should both be monitored.	The same real adjustment applied to another variable. CFRN does not measure the fiscal stance or primary balance.
Bléjer and Cheasty (1991)	Which fiscal concept fits which question? They compare cash/accrual, perimeters, quasi-fiscal operations, and assets.	No single fiscal measure is valid for every purpose; the definition must follow the analytical objective.	Methodological precedent for a dedicated metric with explicit perimeter and valuation rules.
Effective real rate (IMF practice)	What real cost does the outstanding portfolio bear? Interest over lagged/average debt, adjusted for inflation.	It differs from the marginal rate on new issuance and passes market changes through only gradually.	Approximately equivalent to TCFRN in the simple case; CFRN generalizes composition, netting, and the burden relative to GDP.
Reinhart–Sbrancia (2011, 2015)	How much debt is 'liquidated' through negative real rates and financial repression? Contractual real rate × stock.	Repression plus inflation generated material equivalent savings, often 1–5% of GDP in 1945–1980.	Closest antecedent to negative CFRN/GDP. CFRN is symmetric and does not require financial repression.
Hall–Sargent (2011)	What economic return did debt generate and how did it affect debt/GDP? Maturity-specific holding-period returns.	Official interest and market return are different concepts; capital gains and losses matter.	Provides the foundation for a market-value CFRN-M, kept separate from contractual CFRN-C.
Hilscher–Raviv–Reis (2022)	How much debt could future inflation debase? Maturities, holders, and risk-adjusted inflation distributions.	Inflation alone probably has limited capacity to debase large amounts of U.S. debt under plausible scenarios.	Prospective and market-based. Current CFRN is ex post and periodic.

3.8. Synthesis: where the CFRN contribution lies

The preceding literature contains nearly all of the individual mechanisms used by CFRN, but organizes them around different questions. Buiter studies public wealth; the operational deficit studies the fiscal position; the effective real rate measures the average cost of the portfolio; Reinhart and Sbrancia focus on liquidation episodes under negative real rates; Hall and Sargent study market returns and debt-to-GDP dynamics; and Hilscher, Raviv, and Reis assess prospectively the ability of future inflation to erode debt.

The defensible contribution of CFRN is not a new algebraic identity linking interest and inflation. It is the proposal of a periodic, symmetric, replicable convention for measuring the sovereign's net real financial cost, with two interconnected relative outputs—TCFRN and CFRN/GDP—explicit rules for stock-flow consistency, indexation, currency, assets, and consolidation, and a formal separation between contractual and market-value bases. This formulation turns existing building blocks into a measurement standard that can be compared across years and countries.

4. Methodological Principles

4.1. Priority of relative indicators

The absolute monetary value of CFRN depends on the size of the economy and the currency unit and is therefore of limited value for international comparison. The framework treats TCFRN and CFRN/GDP as the principal outputs. The absolute amount is retained for accounting reconciliation and decomposition.

4.2. Stock-flow consistency

The most important methodological rule is that numerator and denominator must refer to the same financial universe. If interest flows include indexed debt, loans, deposits, or foreign-currency debt, the stock used to calculate the rate must include those same liabilities—unless both flow and stock are excluded together. Dividing interest on the entire debt portfolio by an 'inflation-erodible debt' subset mechanically biases the rate.

4.3. Accrual versus cash accounting

The benchmark version should use accrued interest, not cash payments alone. Accrual accounting assigns the financing cost to the period in which it is generated and correctly handles zero-coupon bonds, securities issued at discounts or premiums, and other instruments whose economic cost does not coincide with the date of payment. GFSM and ESA statistics provide the preferred framework. A cash version may be reported as a complement when the objective is budgetary liquidity.

4.4. Contractual or accounting CFRN (CFRN-C): benchmark and historical basis

Contractual or accounting CFRN (CFRN-C) measures the accrued real cost of financing under the contractual obligations actually in force. It uses accrued carry, contractual adjustments to principal—such as price or currency indexation—and a liability base at nominal, face, or accounting value consistent with those flows. It does not treat market-price movements caused by changes in the yield curve as current financing costs when those movements do not alter the issuer's contractual obligation.

CFRN-C is the recommended variant for the main historical series and for international fiscal comparison. Its inputs can be reconstructed over long periods from public accounts, debt statistics, and inflation data; it also retains a direct connection with budgetary interest and the portfolio's effective rate. A creditor's mark-to-market loss is not recorded as a contractual 'saving' for the Treasury unless it produces an actual modification of the liability—through repurchase, restructuring, contractual indexation, or another mechanism recognized in the measurement base.

4.5. Market-value CFRN (CFRN-M): a valuation extension

Market-value CFRN (CFRN-M) answers a different question: what was the real holding-period return on public debt and, symmetrically, how did the marked-to-market economic value of the sovereign's liability change? For an individual security, the nominal holding-period return includes the coupon and the change in market price. For an aggregate portfolio, the calculation requires instrument-level prices or market values and an adjustment for issuance, redemptions, and other flows so that new transactions are not confused with revaluations.

$$h^{M}_{t} = (\text{coupon}_t + Q_t - Q_{t-1}) / Q_{t-1}$$

$$\text{TCFRN}^{M}_{t} = (1 + h^{M}_{t})/(1 + \pi_t) - 1$$

$$\text{CFRNGDP}^{M}_{t} = \text{TCFRN}^{M}_{t} \cdot (\bar{L}^{M}_{t} / \text{GDP}_t)$$

A fall in bond prices generates a loss for creditors and reduces the market value of the liability, so it can lower CFRN-M. But the government still owes the contractual principal unless it repurchases or restructures the security. CFRN-M therefore does not replace CFRN-C: it is a complementary extension useful for valuation, duration, interest-rate risk, and comparison with work such as Hall and Sargent. Reporting both variants is advisable only when methods and data permit them to be clearly separated.

4.6. Ex post versus prospective

The indicator is calculated using observed inflation and interest. It is not designed to value how much debt might be debased by future inflation. Expectations matter indirectly because they affect nominal rates and therefore observed financing costs. Unexpected inflation and forward-looking scenarios should be analyzed through complementary modules.

5. Notation and Measurement Perimeter

The framework defines a base of interest-bearing financial liabilities consistent with the interest expenditure being measured. The recommended institutional unit for international comparison is consolidated general government. For country-specific analysis, central government or the Treasury may be used, provided that the perimeter is stated explicitly.

Table 2. Main notation

Symbol	Definition
$L_{j,t}$	Average stock of interest-bearing financial liability category j included within the perimeter.
$A_{k,t}$	Average stock of interest-bearing financial asset category k included in the net variant.
$c_{j,t}$	Effective nominal carry rate on liability j in its contractual unit, excluding principal revaluation from indexation or currency movements.
$y_{k,t}$	Effective nominal return on asset k, using the same convention.
$v_{j,t}$	Nominal rate of change in principal of liability j due to contractual indexation or exchange-rate movements.
$v^{A}{}_{k,t}$	Nominal rate of change in principal of asset k due to contractual indexation or exchange-rate movements.
π_t	Inflation according to the selected deflator during the period.
$r^{L}{}_{j,t}$	Ex post real cost of liability j.
$r^{A}{}_{k,t}$	Ex post real return on asset k.
$\bar{L}_t$	Total average base of included interest-bearing financial liabilities.
$TCFRN_t$	Net Real Financial Cost Rate.
$CFRNGDP_t$	Net Real Financial Cost as a proportion of GDP.

6. Formulation: From the Simple Case to the General Framework

6.1. Simple case: domestic nominal debt

If the entire liability base is nominal, denominated in domestic currency, the principal is not indexed, and the stock remains approximately stable over the period, the effective nominal carry rate can be written as:

$$i_t = IE_t / \bar{L}_t$$

and the ex post effective real rate, under a one-period Fisher convention, is:

$$r_t = (1 + i_t) / (1 + \pi_t) - 1$$

For moderate inflation and interest rates:

$$r_t \approx i_t - \pi_t$$

This identity is the simplest special case of TCFRN. It should not be presented as an algebraic innovation.

6.2. Liabilities with indexation or foreign-currency denomination

For a liability j whose principal changes nominally over the period at rate ν_j—because of contractual indexation or foreign-currency conversion—and whose effective nominal carry in the instrument's contractual unit is c_j, a one-period representation is:

$$r^L_{j,t} = [(1 + \nu_{j,t})(1 + c_{j,t})] / (1 + \pi_t) - 1$$

This formula nests several cases. For domestic nominal debt, $\nu = 0$. For debt indexed to an index that approximately matches the chosen deflator, $\nu \approx \pi$ and inflation does not erode the real principal. For foreign-currency debt, ν captures the change in the domestic-currency price of one unit of foreign currency. If the domestic currency depreciates by as much as domestic prices rise, the inflationary advantage on that principal may disappear.

Floating-rate debt does not require an artificial reduction in the principal's exposure to inflation. If the principal remains nominal, $\nu = 0$. Rate resets are reflected in c_j. Likewise, short-term debt remains exposed to inflation while it is outstanding; its short duration mainly affects the speed at which nominal financing costs reset upon refinancing.

6.3. Aggregation across liabilities

The real cost of the liability portfolio is a stock-weighted average:

$$TCFR^L_t = \Sigma_j (L_{j,t} / \bar{L}_t) \cdot r^L_{j,t}$$

where $\bar{L}_t = \Sigma_j L_{j,t}$. This is the real cost rate on liabilities before incorporating public financial assets.

6.4. Net financial-asset variant

If 'net' is intended to mean more than subtracting interest income and instead to represent the financial position symmetrically, government nominal assets must also be adjusted for inflation, indexation, and currency effects. For each asset k:

$$r^A_{k,t} = [(1 + \nu^A_{k,t})(1 + y_{k,t})] / (1 + \pi_t) - 1$$

The fully balance-sheet-based TCFRN, normalized by the gross liability base, is defined as:

$$TCFRN_t = [\Sigma_j r^L_{j,t} L_{j,t} - \Sigma_k r^A_{k,t} A_{k,t}] / \bar{L}_t$$

This normalization preserves a positive, comparable base even when government holds substantial assets. Here, 'net' means that the real returns on interest-bearing financial assets reduce the public sector's financing cost. Equity holdings, participations, and other risky assets are excluded unless an explicit public balance-sheet extension is constructed, because including them would transform a debt-cost indicator into a general measure of portfolio profitability.

6.5. The identity linking TCFRN, CFRN/GDP, and the scale of liabilities

TCFRN and CFRN/GDP are not independent indicators. When both use the same net real financial cost as the numerator and TCFRN is normalized by the same average liability base $\bar{L}$, an exact accounting identity follows. By definition:

$$TCFRN_t = CFRN_t / \bar{L}_t$$

Multiplying both sides by $\bar{L}_t/GDP_t$:

$$TCFRN_t \cdot (\bar{L}_t / GDP_t) = (CFRN_t / \bar{L}_t) \cdot (\bar{L}_t / GDP_t)$$

Cancelling $\bar{L}_t$ on the right-hand side gives:

$$TCFRN_t \cdot (\bar{L}_t / GDP_t) = CFRN_t / GDP_t = CFRNGDP_t$$

Therefore:

$$CFRNGDP_t = TCFRN_t \cdot (\bar{L}_t / GDP_t)$$

Although the proof is elementary, the identity has an important economic interpretation: it decomposes the real financing burden into a price component and a scale component. TCFRN is the effective real price of financing per unit of liability; $\bar{L}$/GDP measures the scale of financial exposure relative to the size of the economy; their product is the real macroeconomic burden. Two countries can face the same TCFRN but very different burdens if their liability ratios differ, or the same burden with different combinations of cost and indebtedness.

The same relationship also permits a direct comparison between the nominal interest burden and the real burden. If $i^{eff}_t = NI_t/\bar{L}_t$, then:

$$NI_t / GDP_t = i^{eff}_t \cdot (\bar{L}_t / GDP_t)$$

and the gap between the nominal budgetary indicator and CFRN/GDP is:

$$Gap_t = (i^{eff}_t - TCFRN_t) \cdot (\bar{L}_t / GDP_t)$$

In the simple case of domestic nominal debt, with TCFRN = $(1+i)/(1+\pi) - 1$, the exact difference between the effective nominal rate and TCFRN is $[(1+i)\pi]/(1+\pi)$, which for moderate rates is approximately π. Consequently, the nominal-real gap relative to GDP is approximately inflation multiplied by the liabilities-to-GDP ratio. This decomposition is useful for historical analysis because it separates three sources of variation: the effective nominal cost, inflation, and the scale of indebtedness.

6.6. Absolute CFRN as a derived quantity

Only after calculating the rate is it useful, where needed, to define an auxiliary monetary amount:

$$CFRN_t = TCFRN_t \cdot \bar{L}_t$$

To express this amount in 'real euros' or another constant currency, the base period of the deflator must be specified. For international comparison, TCFRN and CFRN/GDP are preferable because they avoid arbitrary differences of scale and currency.

6.7. Complementary market-value formulation

Where sufficiently detailed valuation data are available, a separate market-value version can be estimated. Let HPR_t denote the aggregate nominal holding-period return on the government debt portfolio, calculated on market values and adjusted for net issuance, redemptions, and other flows. The real market rate is defined as:

$$TCFRN^{M}_{t} = (1 + HPR_t)/(1 + \pi_t) - 1$$

and its macroeconomic burden as:

$$CFRNGDP^{M}_{t} = TCFRN^{M}_{t} \cdot (\bar{L}^{M}_{t} / GDP_t)$$

The price × scale identity remains valid, but the interpretation changes: $\bar{L}^{M}$ is average market value and the numerator incorporates revaluations. CFRN-C and CFRN-M should not be merged into a single series. For long historical series, CFRN-C is recommended as the primary standard; CFRN-M should be presented as an additional module where price and maturity data permit robust estimation.

7. Operational Version Using Aggregate Data

In many countries, publicly available data are not sufficiently detailed to reconstruct c_j, ν_j, and average stocks instrument by instrument. A first-level operational version is therefore proposed, provided that the composition of debt does not make the simplification materially misleading.

If most liabilities are domestic nominal liabilities and interest receipts are small, one may estimate:

$$i^{eff}_{t} = \text{accrued interest / average interest-bearing liabilities}$$

$$TCFRN_t \approx i^{eff}_{t} - \pi_t$$

$$CFRNGDP_t \approx TCFRN_t \cdot (\text{average interest-bearing liabilities} / GDP_t)$$

If net interest—interest paid minus interest received—is used, the result should be explicitly labeled the 'net-flow variant' while retaining gross liabilities in the denominator. When financial assets are material, the full balance-sheet variant should also be published or, at a minimum, interest receipts and nominal financial-asset stocks should be shown separately.

The simplification $i - \pi$ should not be applied without adjustment to portfolios containing a large share of indexed or foreign-currency debt. In such cases, the category-by-category formulation is preferable.

8. A Critical Accounting Issue: Indexed Debt

Debt linked to price indices requires special treatment because national-accounts and public-finance standards may record part of principal indexation as interest. Under GFSM 2014 and ESA 2010, when the principal of an instrument is linked to a broad price index, the increase attributable to that index may be recorded as accrued interest and reinvested in the instrument.

If that component is retained in full in the numerator while the indexed principal is simultaneously treated as immune to inflation erosion, CFRN will overstate the real cost. The solution is to separate financial carry from contractual revaluation of principal: the principal-indexation component should enter ν and must not be counted a second time as real carry.

In practical terms, every empirical application must document whether its interest source includes principal indexation. This check is mandatory in countries where inflation-linked bonds represent a material share of debt.

9. Foreign-Currency Debt and the Real Exchange Rate

Treating all foreign-currency debt as 'non-erodible' is an excessive simplification. Its real burden in domestic currency depends jointly on domestic inflation and exchange-rate movements. If s denotes the domestic-currency price of foreign currency, the nominal revaluation of principal is approximately $\Delta s/s$ and enters through ν.

With 5 percent domestic inflation and a stable exchange rate, the domestic real value of a foreign obligation expressed in local currency declines. A 5 percent nominal depreciation can approximately offset that effect. A larger depreciation can increase the real value of the liability. The relevant variable is therefore the movement of principal in real terms, not a binary domestic/foreign classification.

10. Time Frequency, Average Debt, and Refinancing

Applying annual inflation to an end-of-period stock is a weak approximation when the stock changes rapidly. The recommendation is to calculate the indicator quarterly or monthly and aggregate the real costs of each subperiod. This assigns to each liability only the inflation experienced while it remains on the balance sheet.

With monthly data, the real subperiod cost can be calculated for each category and weighted by the month's average stock. The annual rate is then obtained by aggregating those costs, avoiding the attribution of twelve months of inflation to debt issued near the end of the year.

When only annual data are available, the average of opening and closing stocks may be used as an approximation, but it should be labeled as such. In systems with large concentrated issuance or high inflation, the higher-frequency version is methodologically preferable.

Refinancing should not be adjusted twice. If a bill matures and another is issued at a higher rate, the new rate enters observed carry. Short maturity does not eliminate the real erosion of principal during the period in which each bill was outstanding; it simply accelerates the pass-through of expected inflation and market rates into interest expenditure.

11. Choice of Deflator

There is no uniquely correct real numeraire. The choice should follow the purpose of the indicator and be transparent. For CFRN/GDP, the GDP deflator is a particularly coherent benchmark because it expresses cost in units of the purchasing power of domestic output and aligns with aggregate fiscal capacity. For analysis of holders' purchasing power or comparison with historical literature, CPI/HICP may be preferable. In the United States, PCE can be added as an institutionally relevant sensitivity measure.

The recommended reporting convention is:

- Primary indicator: GDP deflator, when the objective is fiscal burden relative to GDP.
- Sensitivity 1: headline CPI/HICP.
- Sensitivity 2 for the United States: headline PCE.

Core inflation and moving averages should not be presented as alternative deflators for the real cost actually observed. They may be used as clearly labeled 'structural' or smoothed analytical indicators, but they do not represent the period's total loss of purchasing power.

12. Institutional Perimeter and the Central Bank

For international comparison, the benchmark perimeter should be consolidated general government, in line with SNA/ESA/GFSM. Flows and stocks among included subsectors are eliminated. The central bank, however, is not part of general government in national accounts; Treasury debt held on its balance sheet therefore remains a liability of general government.

If a consolidated public-sector variant including the central bank is constructed, Treasury securities held by the central bank should be cancelled as an intra-public-sector position, but the central-bank liabilities financing that position must be included—especially interest-bearing reserves and, if the concept is broadened, monetary liabilities. Failing to do so can create a spurious reduction in consolidated financing cost.

For this reason, where relevant, two separate perimeters should be reported: general government and the consolidated public sector including the central bank. Interest from one perimeter must never be combined with the debt stock of the other.

13. Relationship to Debt-to-GDP Dynamics

CFRN/GDP should be situated within standard debt arithmetic to avoid overinterpretation. In a simplified discrete formulation, abstracting from additional valuation effects and expressing the primary balance as a deficit, one may write:

$$b_t - b_{t-1} = [(r_t - g_t)/(1 + g_t)]\cdot b_{t-1} + d_t + sf_t$$

where b is debt/GDP, r is the effective real interest rate on debt, g is real growth, d is the primary deficit/GDP, and sf denotes stock-flow adjustments. For moderate rates, the real-interest component is approximately r·b and the growth effect approximately −g·b. In the simple case with no assets and a compatible debt base, CFRN/GDP represents precisely the real financing-cost component of this dynamic, while the full equation also includes growth, the primary balance, and stock-flow adjustments.

This relationship makes two points clear. First, CFRN/GDP is not an alternative theory of sustainability, but an explicit way of reporting one component of standard debt dynamics. Second, the real erosion of the liability should not be confused with the denominator effect through which inflation raises nominal GDP. Hall and Sargent study these channels jointly; CFRN seeks to isolate the real financing cost.

14. Observed and Unexpected Inflation

The basic TCFRN is ex post and uses realized inflation. To analyze unanticipated redistribution, a surprise variant can be constructed. A multiplicative measure of the inflation surprise is:

$$\pi^u_t = (1 + \pi_t)/(1 + \pi^e_t) - 1$$

where π^e is expected inflation at the start of the period. This variable can be used to estimate how much of the debtor's real gain came from inflation not embedded in nominal contract terms. It should remain separate from total TCFRN, because fully anticipated inflation is still part of ex post real cost even if its compensation is reflected in coupons or refinancing rates.

15. Minimum Reporting Protocol

Every applied calculation should be accompanied by a methodological note that makes it reproducible. At a minimum, the following should be reported:

1. Institutional perimeter and degree of consolidation.
2. Accounting basis: accrual or cash, with accrual preferred.
3. Definition of interest paid and received, and treatment of indexation.
4. Liability base used and temporal averaging method.
5. Share of inflation-linked debt and adjustment criterion.
6. Share of foreign-currency debt and treatment of the exchange rate.
7. Primary deflator and sensitivity results.
8. TCFRN and CFRN/GDP as the main outputs.
9. Effective nominal rate and liabilities/GDP as diagnostic components.
10. If a net variant is reported, the stocks and returns of the included financial assets.

Table 3. Recommended results template

Variable	Result	Function
Gross interest / GDP	—	Nominal budgetary pressure.
Net interest / GDP	—	Budgetary pressure after interest income.
Effective nominal rate	—	Nominal carry of the outstanding portfolio.
Deflator used	—	Conversion into real terms.
TCFRN	—	Primary financial indicator.
Interest-bearing liabilities / GDP	—	Scale of exposure.
CFRN / GDP	—	Primary macroeconomic indicator.
CPI/HICP or PCE variant	—	Sensitivity.
Indexed / FX / floating	—	Composition diagnosis, not a substitute for the formula.

16. Consistency Tests and Boundary Cases

A methodological indicator should pass simple consistency tests:

- Zero inflation: with $\nu = 0$, real cost equals the effective nominal cost.
- Homogeneous domestic nominal debt: TCFRN reduces to the Fisher effective real rate and, approximately, to $i - \pi$.
- Debt fully indexed to the same deflator: inflation does not erode the real principal; real cost depends on real carry, provided that principal indexation is not duplicated within interest.
- Foreign-currency debt with nominal depreciation equal to domestic inflation: principal approximately preserves its domestic real value.
- Floating rate: nominal principal remains exposed to inflation; the coupon reset appears in the observed nominal rate.
- Negative TCFRN: no zero floor is applied. A negative sign is a valid outcome for the real holding cost, even though it does not imply a budget surplus or a social gain.

- Inconsistent perimeter: if interest refers to a broader universe than the stock, the indicator should be rejected until the two bases are reconciled.

17. Numerical Examples

17.1. Homogeneous case

Suppose average domestic nominal liabilities equal 100 percent of GDP, the effective nominal rate is 3 percent, and inflation is 4 percent. The linear approximation gives:

$$\text{TCFRN} \approx 3\% - 4\% = -1\%$$

$$\text{CFRN/GDP} \approx -1\% \times 100\% = -1\% \text{ of GDP}$$

Using the exact Fisher transformation, the rate is approximately −0.96 percent. The difference is small at these rates and increases at higher inflation rates.

17.2. Same rate, different scale

If another government records the same TCFRN but its liability base equals 150 percent of GDP, CFRN/GDP will be approximately −1.5 percent. The identity CFRN/GDP = TCFRN × liabilities/GDP shows that the difference does not arise from the real 'price' of financing—which is the same—but from the scale of exposure. Conversely, two countries with the same debt-to-GDP ratio can face different real burdens solely because their TCFRNs differ.

17.3. Indexed debt

Suppose a bond's principal rises by 4 percent because it is price-indexed and its contractual real carry is 1 percent. With 4 percent inflation, principal revaluation offsets the erosion and the real cost remains around 1 percent. Treating the 4 percent indexation as additional real interest while also assigning zero erosion to principal would double-count the same effect.

17.4. Foreign-currency debt

With 4 percent domestic inflation, dollar-denominated debt, and 6 percent domestic-currency depreciation, the local-currency principal rises faster than the price level. Foreign debt can therefore become more burdensome in real terms despite inflation. This example shows why a binary 'erodible/non-erodible' coefficient is inferior to a formulation that explicitly incorporates changes in principal.

18. Sources of Error and Controls

Table 4. Main methodological risks

Risk	Bias	Control
Interest on all debt / partial stock	Artificially high effective rate	Mandatory stock-flow consistency.
Principal indexation recorded as interest	Overstates the real cost of indexed debt	Separate carry from contractual indexation.
Foreign debt treated as λ=0	Ignores real exchange-rate effects	Incorporate nominal revaluation of principal.

Risk	Bias	Control
Discounting short-term debt because of duration	Understates current erosion	Use high-frequency data; refinancing enters through carry.
Cash instead of accrual	Shifts cost across periods	Use accrued D.41/GFSM interest where possible.
Netting income without assets	Balance-sheet asymmetry	Publish the flow-net and balance-sheet variants separately.
Core or moving average as 'real inflation'	Confuses observed cost with a structural signal	Reserve them for sensitivity/smoothed indicators.
Mixing general government and central bank	Double counting or false cancellation	Declare the perimeter and reconstruct consolidated liabilities.
Negative CFRN = overall fiscal improvement	Overinterpretation	Separate financing cost, primary balance, and growth.

19. Hypotheses for the Applied Comparative Paper

The methodology generates concrete empirical hypotheses that can be tested in a second stage:

- Country rankings by interest/GDP will differ from rankings by CFRN/GDP, especially during episodes of heterogeneous inflation.
- The gap between effective nominal cost and TCFRN will be larger in countries with more unindexed domestic nominal debt.
- Short- or floating-rate debt will not necessarily reduce current erosion, but it will transmit expected inflation and market rates more rapidly into nominal carry.
- Countries with a high share of inflation-linked debt will show a smaller improvement in TCFRN during inflation shocks once principal indexation is removed from recorded interest.
- Foreign-currency debt can generate very different results depending on movements in the real exchange rate.
- A consolidated measure including the central bank will differ from the general-government measure when interest-bearing reserves economically replace Treasury debt held by the central bank.
- A favorable CFRN/GDP in one year can coexist with a subsequent deterioration in the effective cost as the portfolio is refinanced at higher nominal rates.

20. Discussion: Usefulness and Limits

CFRN is primarily a descriptive and comparative tool. It forces three pieces of information that are often reported separately into a single analytical architecture: the effective cost of the portfolio, the change in the real value of principal, and the scale of liabilities relative to GDP. By prioritizing TCFRN and CFRN/GDP, it distinguishes an economy with little but expensive debt from one with very large debt that is temporarily cheap in real terms.

Its main limitation is that it cannot replace a full decomposition of debt dynamics. Nor is it a measure of market value, refinancing risk, or intertemporal sustainability. Contractual composition and statistical quality may require difficult adjustments, especially for indexed bonds, hybrid instruments, and debt denominated in multiple currencies.

The net-asset variant also raises a normative perimeter choice. If only interest-bearing nominal assets are included, the indicator remains a financing-cost measure. If equities, public enterprises, or real assets are added, the metric ceases to be directly comparable with a financing cost and moves toward a public-sector portfolio income statement. The extension should therefore remain tightly bounded.

21. Recommended Final Definition

The central definition of the framework is a contractual net real financing rate—CFRN-C—calculated over a base of interest-bearing financial liabilities consistent with the interest flows and adjusted for the contractual real change in principal for each category. The two recommended reporting indicators are:

$$TCFRN_t = [\Sigma_j r^L_{j,t} L_{j,t} - \Sigma_k r^A_{k,t} A_{k,t}] / \bar{L}_t$$

$$CFRNGDP_t = TCFRN_t \cdot (\bar{L}_t / GDP_t)$$

with:

$$r^L_{j,t} = [(1 + \nu_{j,t})(1 + c_{j,t})]/(1 + \pi_t) - 1$$

and an analogous expression for included assets. In the simple case of domestic nominal debt with no material financial assets:

$$TCFRN_t = (1 + i_t)/(1 + \pi_t) - 1 \approx i_t - \pi_t$$

$$CFRNGDP_t = TCFRN_t \cdot (\bar{L}_t / GDP_t)$$

The monetary CFRN amount is then derived as TCFRN × $\bar{L}$ and should not displace the two relative measures as the principal outputs.

The market-value variant, CFRN-M, should be identified explicitly and kept outside the main contractual series:

$$TCFRN^M_t = (1 + HPR_t)/(1 + \pi_t) - 1 \quad ; \quad CFRNGDP^M_t = TCFRN^M_t \cdot (\bar{L}^M_t / GDP_t)$$

CFRN-C measures the sovereign's contractual/accrued real cost and is the recommended basis for historical series. CFRN-M measures the portfolio's real market return and the marked-to-market change in the liability. Both are economically meaningful, but they answer different questions and should not be conflated.

22. Conclusion

The nominal budgetary cost of debt and its real economic cost are different quantities. The former measures debt-service flows; the latter must also account for what happens to the real value of principal while the debt remains on the balance sheet. This intuition is well established in the literature on operational deficits, effective real rates, debt liquidation, and debt-to-GDP dynamics.

The methodological contribution of CFRN should therefore be stated precisely. It does not propose a new identity between interest and inflation. It proposes a measurement and reporting standard centered on two outcomes: TCFRN, the real cost per unit of liability, and CFRN/GDP, the real burden on the economy.

They are linked by the identity CFRN/GDP = TCFRN × liabilities/GDP, which transparently separates the real price of financing from the scale of indebtedness.

The framework strengthens consistency through stock-flow matching, accrual accounting, separation of carry from principal indexation, explicit exchange-rate treatment, prudent netting of financial assets, and institutional consolidation rules. It also formally distinguishes CFRN-C—contractual/accounting and appropriate for the main historical series—from CFRN-M—market-based and built on holding-period returns—thereby avoiding the mistake of treating a mere decline in bond prices as a contractual saving for the Treasury.

In its simplest form, the indicator coincides with the effective real rate already used in macrofiscal analysis and with the logic underlying the 'liquidation effect' in years of negative real rates. Its potential value added lies in generalizing that logic across all signs, turning it into a stable pair of relative indicators, and providing a replicable convention for heterogeneous public-debt portfolios. The natural next step is to apply the protocol to a cross-country sample and assess how much it changes the conventional diagnosis based on nominal interest relative to GDP.

References

Bléjer, M. I., & Cheasty, A. (1991). The Measurement of Fiscal Deficits: Analytical and Methodological Issues. Journal of Economic Literature, 29(4), 1644–1678.

Bléjer, M. I., & Cheasty, A. (Eds.). (1991). How to Measure the Fiscal Deficit: Analytical and Methodological Issues. Washington, DC: International Monetary Fund.

Buiter, W. H. (1983). Measurement of the Public Sector Deficit and Its Implications for Policy Evaluation and Design. IMF Staff Papers, 30(2), 306–349.

Congressional Budget Office. (2020). Federal Net Interest Costs: A Primer. Washington, DC: CBO.

European Commission / Eurostat. (2019). Manual on Government Deficit and Debt: Implementation of ESA 2010. Luxembourg: Publications Office of the European Union.

Hall, G. J., & Sargent, T. J. (2011). Interest Rate Risk and Other Determinants of Post-WWII U.S. Government Debt/GDP Dynamics. American Economic Journal: Macroeconomics, 3(3), 192–214. https://doi.org/10.1257/mac.3.3.192

Hilscher, J., Raviv, A., & Reis, R. (2022). Inflating Away the Public Debt? An Empirical Assessment. The Review of Financial Studies, 35(3), 1553–1595. https://doi.org/10.1093/rfs/hhab018. Working-paper version: NBER Working Paper 20339 (2014).

International Monetary Fund. (2014). Government Finance Statistics Manual 2014. Washington, DC: International Monetary Fund.

International Monetary Fund. (2023). World Economic Outlook, April 2023: A Rocky Recovery. Box 3.2, Monetary and Fiscal Interactions. Washington, DC: International Monetary Fund.

Reinhart, C. M., & Sbrancia, M. B. (2011). The Liquidation of Government Debt. BIS Working Papers No. 363. Bank for International Settlements.

Reinhart, C. M., & Sbrancia, M. B. (2015). The Liquidation of Government Debt. IMF Working Paper WP/15/7. Washington, DC: International Monetary Fund.

Tanzi, V., Blejer, M. I., & Teijeiro, M. O. (1987). Inflation and the Measurement of Fiscal Deficits. IMF Staff Papers, 34(4), 711–738.